Field Theory Models without the Cosmological Constant Problem

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Abstract

We study field theory models in the context of a gravitational theory without the cosmological constant problem (CCP). The theory is based on the requirement that the measure of integration in the action is not necessarily $\sqrt{-g}$ but it is determined dynamically through additional degrees of freedom, like four scalar fields φ_a. We study three possibilities for the general structure of the theory: (A) The total action has the form $S = \int \Phi L d^4x$ where the measure Φ is built from the scalars φ_a in such a way that the transformation $L \rightarrow L + const$ does not effect equations of motion. Then an infinite dimensional shifts group of the measure fields (SGMF) φ_a by arbitrary functions of the Lagrangian density

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L $\varphi_a \to \varphi_a + f_a(L)$ is recognized as the symmetry group of the action up to an integral of a total divergence. (B) The total action has the form $S = S_1 + S_2$, $S_1 = \int \Phi L_1 d^4x$, $S_2 = \int \sqrt{-g} L_2 d^4x$ which is the only model different from (A) and invariant under SGMF (but now with $f_a = f_a(L_1)$). Similarly, now only S_1 satisfies the requirement that the transformation $L_1 \to L_1 + const$ does not effect equations of motion. Both in the case (A) and in the case (B) it is assumed that L, L_1, L_2 do not depend on φ_a. (C) The action includes a term which breaks the SGMF symmetry. It is shown that in the first order formalism in cases (A) and (B) the CCP is solved: the effective potential vanishes in a true vacuum state (TVS) without fine tuning. We present a few explicit field theory models where it is possible to combine the solution of the CCP with: 1) possibility for inflationary scenario; 2) spontaneously broken gauge unified theories (including fermions). In the case (C), the breaking of the SGMF symmetry induces a nonzero energy density for the TVS. When considering only a linear potential for a scalar field ϕ in S_1, the continuous symmetry $\phi \to \phi + const$ is respected. Surprisingly, in this case SSB takes place while no massless ("Goldstone") boson appears. We discuss the role of the SGMF symmetry in the possible connection of this theory with theories of extended objects.

1 Introduction

The cosmological constant problem in the context of general relativity (GR) can be explained as follows. In GR one can introduce such a constant or one may set it to zero. The problem is that after further investigation of elementary particle theory, we discover new phenomena like radiative corrections, the existence of condensates, etc. each of which contributes to the vacuum energy. In order to have a resulting zero or extremely small cosmological constant as required by observations of the present day universe , one would have to carefully fine tune parameters in the Lagrangian so that all of these contributions more or less exactly cancel. This question has captured the attention of many authors because, among other things, it could be a serious indication that something fundamental has been missed in our standard way of thinking about field theory and the way it must couple to gravity. For a review of this problem see [1].

The situation is made even more serious if one believes in the existence of an inflationary phase for the early universe, where the vacuum energy plays an essential role. The question is then: what is so special about the present vacuum state which was not present in the early universe? In this paper we are going to give an answer this question.

As it is well known, in nongravitational physics the origin from which we measure energy is not important. For example in nonrelativistic mechanics a shift in the potential $V \rightarrow V + constant$ does not lead to any consequence in the equations of motion. In the GR the situation changes dramatically. There *all* the energy density,

including the origin from which we measure it, affects the gravitational dynamics.

This is quite apparent when GR is formulated from a variational approach. There the action is

$$S = \int \sqrt{-g} L d^4 x \tag{1}$$

$$L = -\frac{1}{\kappa} R(g) + L_m \tag{2}$$

where $\kappa = 16\pi G$, $R(g)$ is the Riemannian scalar curvature of the 4-dimensional space-time with metric $g_{\mu\nu}$, $g \equiv Det(g_{\mu\nu})$ and L_m is the matter Lagrangian density. It is apparent now that the shift of the Lagrangian density L, $\quad L \rightarrow L + C, \quad C = const$ is not a symmetry of the action (1). Instead, it leads to an additional piece in the action of the form $C \int \sqrt{-g} d^4 x$ which contributes to the equations of motion and in particular generates a so called "cosmological constant term" in the equations of the gravitational field.

In Refs. [2]-[5] we have developed an approach where the cosmological constant problem is treated as the absence of gravitational effects of a possible constant part of the Lagrangian density. The basic idea is that the measure of integration in the action principle is not necessarily $\sqrt{-g}$ but it is allowed to "float" and to be determined dynamically through additional degrees of freedom. In other words the floating measure is not from first principles related to $g_{\mu\nu}$, although relevant equations will in general allow to solve for the new measure in terms of other fields of the theory ($g_{\mu\nu}$ and matter fields). This theory is based on the demand that such measure respects *the principle of non gravitating vacuum energy* (NGVE principle) which states that the Lagrangian density L can be changed to $L + constant$ without affecting the dy-

namics. This requirement is imposed in order to offer a new approach for the solution of the cosmological constant problem. Concerning the theories based on the NGVE principle we will refer to them as NGVE-theories.

The invariance $L \longrightarrow L + constant$ for the action is achieved if the measure of integration in the action is a total derivative, so that to an infinitesimal hypercube in 4-dimensional space-time $x_0^\mu \leq x^\mu \leq x_0^\mu + dx^\mu$, $\mu = 0, 1, 2, 3$ we associate a volume element dV which is: (i) an exact differential, (ii) it is proportional to d^4x and (iii) dV is a general coordinate invariant. The usual choice, $\sqrt{-g}d^4x$ does not satisfy condition (i).

The conditions (i)-(iii) are satisfied [2], [3] if the measure corresponds to the integration in the space of the four scalar fields $\varphi_a, (a = 1, 2, 3, 4)$, that is

$$dV = d\varphi_1 \wedge d\varphi_2 \wedge d\varphi_3 \wedge d\varphi_4 \equiv \frac{\Phi}{4!} d^4x \qquad (3)$$

where

$$\Phi \equiv \varepsilon_{a_1 a_2 a_3 a_4} \varepsilon^{\mu\nu\lambda\sigma} (\partial_\mu \varphi_{a_1})(\partial_\nu \varphi_{a_2})(\partial_\lambda \varphi_{a_3})(\partial_\sigma \varphi_{a_4}). \qquad (4)$$

Notice that this measure is a particular realization of the NGVE-principle (for other possible realization which leads actually to the same results, see Refs. [4], [5]). For deeper discussion of the geometrical meaning of this realization of the measure see Ref.[6]

We will study three possibilities for the general structure of the action

(A) The most straightforward and complete realization of the NGVE principle

(the so called strong NGVE principle) where the total action is defined as follows

$$S = \int L \Phi d^4 x \tag{5}$$

where L is a total Lagrangian density. We assume in what follows that L does not contain explicitly the measure fields, that is the fields φ_a by means of which Φ is defined.

Introducing independent degrees of freedom related to the measure we arrive naturally at a conception that all possible degrees of freedom that can appear should be considered as such. This is why we expect that the first order formalism, where the affine connection is *not* assumed to be the Christoffel coefficients in general should be preferable to the second order formalism where this assumption is made.

In fact, it is found that the NGVE theory in the context of the first order formalism does indeed provide a solution of the cosmological constant problem [3], while this is not the case when using the second order formalism.

The simplest example [3]-[5] (see also Sec.2) where these ideas can be tested is that of a matter Lagrangian described by a single scalar field with a nontrivial potential. In this case the variational principle leads to a constraint which implies the vanishing of the effective vacuum energy in any possible allowed configuration of the scalar field. These allowed configurations are however constant values at the extrema of the scalar field potential and an integration constant that results from the equations of motion has the effect of exactly canceling the value of the potential at these points. So, the scalar field is forced to be a constant and hence the theory has no nontrivial dynamics for the scalar field.

In this case the measure (4) is not determined by the equations of motion. In fact a local symmetry (called "Local Einstein Symmetry" (LES)) exists which allows us to choose the measure Φ to be of whatever we want. In particular $\Phi = \sqrt{-g}$ can be chosen and in this case the theory coincides in the vacuum with GR with $\Lambda = 0$.

A richer structure is obtained if a four index field strength which derives from a three index potential is allowed in the theory [7],[8]. The introduction of this term breaks the LES mentioned above. In this case, the constraint that the theory provides, allows to solve for the measure Φ in terms of $\sqrt{-g}$ *and* the matter fields of the theory. The equations can be written in a form that resemble those of the Einstein theory by the use of a conformal transformation (or in equivalent language by going to the Einstein frame).

Then the theory which contains a scalar field shows a remarkable feature: the effective potential of the scalar field that one obtains in the Einstein conformal frame is such that generally allows for an inflationary phase which evolves at a later stage, without fine tuning, to a vacuum of the theory with zero cosmological constant [5].

The 4-index field strength also allows for a Maxwell-type dynamics of gauge fields and of massive fermions [8]. An explicit construction of unified gauge theory ($SU(2) \times U(1)$ as an example) based on these ideas and keeping all the above mentioned advantages is presented in Ref. [8].

(B) It is possible to check (see [2]) that the action (5) respects (up to the integral a total divergence) the infinite dimensional group of shifts of the measure fields φ_a

(SGMF)

$$\varphi_a \rightarrow \varphi_a + f_a(L) \tag{6}$$

where $f_a(L)$ is an *arbitrary* differentiable function of the total Lagrangian density L. Such symmetry in general represents a nontrivial mixing between the measure fields φ_a and the matter and gravitational fields (through L). As it was mentioned in Ref. [2], this symmetry prevents the appearance of terms of the form $f(\chi)\Phi$ in the effective action with the single possible exception of $f(\chi) = c/\chi$ where a scalar c is χ independent. This is because in this last case the term $f(\chi)\Phi = c\sqrt{-g}$ is φ_a independent. This possibility gives rise to the cosmological constant term in the action while the symmetry (6) is maintained. This can be generalized to possible contributions of the form $\int L_2\sqrt{-g}d^4x$ where L_2 is φ_a independent function of matter fields and gravity if radiative corrections generate a term $f(\chi)\Phi$ with $f(\chi) = L_2/\chi$.

So, let us consider an action [9] which consists of two terms

$$\begin{aligned} S &= S_1 + S_2 \\ S_1 &= \int L_1 \Phi d^4x \\ S_2 &= \int L_2 \sqrt{-g} d^4x \end{aligned} \tag{7}$$

Now only S_1 satisfies the requirement that the transformation $L_1 \rightarrow L_1 + const$ does not effect equations of motion, which is a somewhat weaker version of the NGVE principle. In this case, the symmetry transformation (6) is replaced by

$$\varphi_a \rightarrow \varphi_a + f_a(L_1) \tag{8}$$

The constraint which appears again in the first order formalism, allows now to

solve the measure Φ in terms of $\sqrt{-g}$ *and* the matter fields of the theory without introducing the four index field strength (see Sec.3).

In scalar field models with potentials entering in S_1 and S_2, in the true vacuum state (TVS) $\chi \to \infty$. However, in the conformal Einstein frame this singularity does not present and the energy density of TVS is zero without fine tuning of any scalar potential in S_1 or S_2 (see Ref.[9]). This means that even the weak version of the NGVE principle is enough to provide a solution of the CCP.

We show in Subsec.3.2 how it is possible to incorporate gauge fields in such kind of model.

(C) As it is well known, in order to really understand the role of some symmetry, one should see what the breaking of such symmetry does. With a simple example in Sec.4, we will see that the breaking of the symmetry (6) or (8) can lead to the appearance of a non zero energy density for the TVS. In the particular example we study, the additional piece in the action that is added is of the form

$$S_3 = -\lambda \int \frac{\Phi^2}{\sqrt{-g}} d^4x. \tag{9}$$

which is equivalent to considering of a piece of the Lagrangian density L_1 linear in χ. Then as we show, the TVS energy density appears to be equal to λ.

In Sec.5 we show that when considering only a linear potential for a scalar field ϕ in S_1, the continuous symmetry $\phi \to \phi + const$ is respected. Surprisingly, in this case SSB takes place while no massless ("Goldstone") boson appears. Models with such a feature exist in each of the cases (A), (B), (C).

2 The strong NGVE principle

Starting from the case (A) (see Introduction), we consider the action $S = \int \Phi L d^4x$. Our choice for the total Lagrangian density is $L = \kappa^{-1} R(\Gamma, g) + L_m$, where L_m is the matter Lagrangian density and $R(\Gamma, g)$ is the scalar curvature $R(\Gamma, g) = g^{\mu\nu} R_{\mu\nu}(\Gamma)$ of the space-time of the affine connection $\Gamma^{\mu}_{\alpha\beta}$: $R_{\mu\nu}(\Gamma) = R^{\alpha}_{\mu\nu\alpha}(\Gamma)$, $R^{\lambda}_{\mu\nu\sigma}(\Gamma) \equiv \Gamma^{\lambda}_{\mu\nu,\sigma} - \Gamma^{\lambda}_{\mu\sigma,\nu} + \Gamma^{\lambda}_{\alpha\sigma}\Gamma^{\alpha}_{\mu\nu} - \Gamma^{\lambda}_{\alpha\nu}\Gamma^{\alpha}_{\mu\sigma}$. This curvature tensor is invariant under the λ-transformation [10] $\Gamma'^{\mu}_{\alpha\beta} = \Gamma^{\mu}_{\alpha\beta} + \delta^{\mu}_{\alpha}\lambda_{,\beta}$. In the NGVE-theory, it allows us to eliminate the contribution to the torsion which appears as a result of introduction of the new measure. However, even after this still there is the non metric contribution to the connection related to the measure (it is expressed in terms of derivatives the scalar field $\chi \equiv \Phi/\sqrt{-g}$).

In addition to this, in the vacuum and in some matter models, the theory possesses a local symmetry which plays a major role. This symmetry consists of a conformal transformation of the metric $g_{\mu\nu}(x) = J^{-1}(x) g'_{\mu\nu}(x)$ accompanied by a corresponding diffeomorphism $\varphi_a \longrightarrow \varphi'_a = \varphi'_a(\varphi_b)$ in the space of the scalar fields φ_a such that $J = Det(\frac{\partial \varphi'_a}{\partial \varphi_b})$. Then for Φ we have: $\Phi(x) = J^{-1}(x)\Phi'(x)$. In the presence of fermions this symmetry is appropriately generalized [3]. For models where it holds, it is possible to choose the gauge where the measure Φ coincides with $\sqrt{-g}$, the measure of GR. This is why we call this symmetry *"local Einstein symmetry"* (LES).

Varying the action with respect to φ_a we get $A^{\mu}_{b}\partial_{\mu}[-\frac{1}{\kappa}R(\Gamma, g) + L_m] = 0$ where $A^{\mu}_{b} = \varepsilon_{acdb}\varepsilon^{\alpha\beta\gamma\mu}(\partial_{\alpha}\varphi_a)(\partial_{\beta}\varphi_c)(\partial_{\gamma}\varphi_d)$. If $Det(A^{\mu}_{b}) = \frac{4^{-4}}{4!}\Phi^3 \neq 0$ then

$$-\frac{1}{\kappa}R(\Gamma, g) + L_m = M = const \tag{10}$$

Performing the variation with respect to $g^{\mu\nu}$ we get (here for simplicity we don't consider fermions)

$$-\frac{1}{\kappa}R_{\mu\nu}(\Gamma)+\frac{\partial L}{\partial g^{\mu\nu}}=0 \tag{11}$$

Contracting eq.(11) with $g^{\mu\nu}$ and making use eq.(10) we get the constraint

$$g^{\mu\nu}\frac{\partial(L_m-M)}{\partial g^{\mu\nu}}-(L_m-M)=0 \tag{12}$$

For the cases where the LES is an exact symmetry, we can eliminate the mentioned above χ-contribution to the connection. Indeed, for $J=\chi$ we get $\chi'\equiv 1$ and $\Gamma'^{\alpha}_{\mu\nu}=\{^{\alpha}_{\mu\nu}\}'$, where $\{^{\alpha}_{\mu\nu}\}'$ are the Christoffel's coefficients corresponding to the new metric $g'_{\mu\nu}$. In this gauge the affine space-time becomes a Riemannian space-time.

When applying the theory to the matter model of a single scalar field with a potential $V(\varphi)$, the constraint (12) implies $V(\varphi)+M=0$, which means that φ is a constant (the equation of motion of φ implies also that $V'(\varphi)=0$). Since $\varphi=constant$, Eq. (11) implies $R_{\mu\nu}(\Gamma,g)=0$ and also Eq. (10) implies $R(\Gamma,g)=0$, if $V(\phi)+M=0$ is taken into account. $V(\phi)+M=0$ dictates the vanishing of the LES violating terms and disappearance of dynamics of φ, so LES is effectively restored on the mass shell. Choosing the gauge $\chi=1$ we see that the potential does not have a gravitational effect since the standard Ricci tensor vanishes and flat space-time remains the only natural vacuum.

The above solution of the CCP is at the price of the elimination of a possible scalar field dynamics. We have shown in [7],[8] that the introduction of a 4-index field strength can restore normal scalar, gauge and fermion dynamics and as a bonus provide: (i) *the possibility of inflation in the early universe with a transition (after*

reheating) to a $\Lambda = 0$ phase without fine tuning and (ii) *a solution to the hierarchy problem in the context of unified gauge dynamics with SSB.*

A four index field strength is derived from a three index gauge potential according to $F_{\mu\nu\alpha\beta} = \partial_{[\mu} A_{\nu\alpha\beta]}$. The physical scenario we have in mind is one where all gauge fields, including $A_{\nu\alpha\beta}$ are treated in a unified way. The possible physical origin of $A_{\nu\alpha\beta}$ can be for example an effective way to describe the condensation of a vector gauge field in some extra dimensions [11]. In this picture for example this means that all gauge fields should appear in a combination having all the same homogeneity properties with respect to conformal transformations of the metric. This is achieved if all dependence on field strengths is through the "gauge fields complex" $y = F^a_{\mu\nu} F^{a\mu\nu} + \frac{\varepsilon^{\mu\nu\alpha\beta}}{\sqrt{-g}} \partial_\mu A_{\nu\alpha\beta}$.

Considering for illustration only one vector gauge field $\tilde{A}_\mu$ and a charged scalar field ϕ, we take a generic action satisfying the above requirements in the unitary gauge ($\phi = \phi^*$; $|\phi| = \frac{1}{\sqrt{2}}\varphi$)

$$S = \int \Phi d^4 x \left[-\frac{1}{\kappa} R(\Gamma, g) - m^4 f(u) + \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - V(\varphi) + \frac{1}{2} \tilde{e}^2 \varphi^2 g^{\mu\nu} \tilde{A}_\mu \tilde{A}_\nu \right], \quad (13)$$

where $u \equiv y/m^4$, m is a mass parameter and $f(u)$ is a nonspecified function which has to have an extremum at some point $u = u_0 > 0$ to provide physically reasonable consequences (see below). This is motivated from the well known instability of non-abelian gauge theories as found by Savvidy [12] and which leads naturally to such type of functions.

The equations of motion obtained from the $A_{\nu\alpha\beta}$ variation imply $\chi f' = \omega =$ *constant* where ω is a dimensionless integration constant. The constraint (12) be-

comes now

$$-2uf'(u)+f(u)+\frac{1}{m^4}[V(\varphi)+M]=0, \qquad (14)$$

which allows to find $u=u(\varphi)$.

One can then see that all equations can be put in the standard GR, scalar field and gauge field form if we make the conformal transformation to the "Einstein frame" $\overline{g}_{\mu\nu}=\chi g_{\mu\nu}$. In the Einstein frame, the scalar field acquires an effective potential

$$V_{eff}(\varphi)=\frac{y}{\omega}\left(f'(u)\right)^2 \qquad (15)$$

If there is a point $u=u_0$ where $f'(u_0)=0$, then if $y_0/\omega>0$, such a state is a stable vacuum of the theory. This vacuum state is defined by the gauge and scalar condensates (u_0,φ_0) connected by the relation $f(u_0)+\frac{1}{m^4}[V(\varphi_0)+M]=0$, representing the exact cancellation of the contributions to the vacuum energy of the scalar field and of gauge field condensate. Therefore the effective cosmological constant in this vacuum becomes zero without fine tuning.

One can see also that $\frac{dV_{eff}}{d\varphi}=\frac{1}{\omega}\frac{df}{du}\frac{dV}{d\varphi}$, so another extremum where $V'=0$ for example can serve as a phase with nonzero effective cosmological constant and therefore inflation becomes possible as well. This vacuum is smoothly connected by dynamical evolution of the scalar field, with the zero cosmological constant phase, thus providing a way to achieve inflation and transition (after standard reheating period) to $\Lambda=0$ phase without fine tuning.

Stability of gauge fields requires $\omega>0$. In this case theory acquires canonical form if the new fields and couplings are defined $A_\mu=2\sqrt{\omega}\tilde{A}_\mu,\ \ e=\frac{\tilde{e}}{2\sqrt{\omega}}$. Appearance of the VEV of the scalar field φ_0 leads to the standard Higgs mechanism.

Fermions can also be introduced [8] in such a way that normal massive propagation is obtained in the Einstein frame. The resulting fermion mass is proportional to $\frac{\varphi_0}{\sqrt{u_0\omega}}$. The gauge boson mass, depending on e also goes as $\propto 1/\sqrt{\omega}$. So we see that a big value of the integration constant ω pushes both effective masses and gauge coupling constants to small values, thus providing a new approach to the solution of the hierarchy problem. Furthermore we see that fermion masses include additional factor $u_0^{-1/2}$. Therefore, if the gauge complex condensate u_0 is big enough, it can explain why fermion masses are much less then boson ones.

Finally, there is no obstacle for the construction of a realistic unified theories (like electroweak, QCD, GUT) along the lines of the simple example displayed above [8]. The common feature of such theories is the fact that the stable vacuum developed after SSB has zero effective cosmological constant.

3 Models satisfying the weak NGVE principle

3.1 The simplest model

To demonstrate how the theory works when it is based on the weak NGVE principle (see the case(B) in Introduction), we start here from the simplest model [9] in the first order formalism including scalar field ϕ and gravity according to the prescription of the NGVE principle and in addition to this we include the standard cosmological constant term. So, we consider an action

$$S = \int L_1 \Phi d^4x + \int \Lambda \sqrt{-g} d^4x \tag{16}$$

where $L_1 = -\frac{1}{\kappa} R(\Gamma, g) + \frac{1}{2} g^{\mu\nu} \phi_{,\mu} \phi_{,\nu} - V(\phi)$.

Performing the variation with respect to the measure fields φ_a we obtain equations $A^{\mu}_a \partial_\mu L_1 = 0$ where $A^{\mu}_b = \varepsilon_{acdb} \varepsilon^{\alpha\beta\gamma\mu} (\partial_\alpha \varphi_a)(\partial_\beta \varphi_c)(\partial_\gamma \varphi_d)$. If $\Phi \neq 0$, then it follows from the last equations that $L_1 = M = const$

Varying the action (16) with respect to $g^{\mu\nu}$ we get

$$\Phi\left(-\frac{1}{\kappa} R_{\mu\nu}(\Gamma) + \frac{1}{2} \phi_{,\mu} \phi_{,\nu}\right) - \frac{1}{2} \sqrt{-g} \Lambda g_{\mu\nu} = 0 \tag{17}$$

Contracting Eq.(17) with $g^{\mu\nu}$ and using equation $L_1 = M$ we obtain the constraint

$$M + V(\phi) - \frac{2\Lambda}{\chi} = 0 \tag{18}$$

where again $\chi \equiv \Phi/\sqrt{-g}$.

The scalar field ϕ equation is $(-g)^{-1/2} \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu \phi) + \sigma_{,\alpha} \phi^{,\alpha} + V' = 0$ where $\sigma \equiv \ln \chi$ and $V' \equiv dV/d\phi$.

The derivatives of the field σ enter both the gravitational equation (17) (through the connection) and in the scalar field equation. By a conformal transformation $g_{\mu\nu} \rightarrow \overline{g}_{\mu\nu} = \chi g_{\mu\nu}; \quad \phi \rightarrow \phi$ to an "Einstein picture" and using the constraint (18) we obtain the canonical form of equations for the scalar field $(-\overline{g})^{-1/2} \partial_\mu (\sqrt{-\overline{g}} \quad \overline{g}^{\mu\nu} \partial_\nu \phi) + V'_{eff}(\phi) = 0$ and the gravitational equations in the Riemannian space-time with metric $\overline{g}_{\alpha\beta}$: $R_{\mu\nu}(\overline{g}_{\alpha\beta}) - \frac{1}{2} \overline{g}_{\mu\nu} R(\overline{g}_{\alpha\beta}) = \frac{\kappa}{2} T^{eff}_{\mu\nu}(\phi)$ where $T^{eff}_{\mu\nu}(\phi) = \phi_{,\mu} \phi_{,\nu} - \frac{1}{2} \overline{g}_{\mu\nu} \phi_{,\alpha} \phi_{,\beta} \overline{g}^{\alpha\beta} + \overline{g}_{\mu\nu} V_{eff}(\phi)$, and $V_{eff}(\phi) = \frac{1}{4\Lambda} [M + V(\phi)]^2$

We see that for any analytic function $V(\phi)$, the effective potential in the Einstein picture has an extremum, i.e. $V'_{eff} = 0$, either when $V' = 0$ or $V + M = 0$. The extremum $\phi = \phi_1$ where $V'(\phi_1) = 0$ has nonzero energy density $[M + V(\phi_1)]^2/4\Lambda$ if

a fine tuning is not assumed. In contrast to this, if $\Lambda > 0$, the state $\phi = \phi_0$ where $V(\phi_0) + M = 0$ is the absolute minimum and therefore ϕ_0 is a true vacuum with zero cosmological constant without any fine tuning. A mass square of the scalar field describing small fluctuations around ϕ_0 is $m^2 = \frac{1}{2\Lambda}[V'(\phi_0)]^2$. Exploiting the possibility to choose any analytic $V(\phi)$, we can pick the structure of $V_{eff}(\phi)$ so that it allows for an inflationary era, the possibility of reheating after scalar field oscillations and the setting down to a zero cosmological constant phase at the later stages of cosmological evolution, without fine tuning.

Notice that if $V + M$ achieves the value zero at some value of $\phi = \phi_0$, this point represents the absolute minimum of the effective potential. If this is not the case for a particular choice of potential V and of integration constant M, it is always possible to choose an infinite range of values of M where this will happen. Therefore no fine tuning of parameters has to be invoked, the zero value of the true vacuum energy density appears naturally in this theory.

Further generalizations, like considering a term of the form $\int U(\phi)\sqrt{-g}d^4x$ instead of $\Lambda \int \sqrt{-g}d^4x$, the possibility of coupling of scalar fields to curvature, etc. do not modify the qualitative nature of the effects described here and they will be studied in a more detailed publication. For example, even in the presence of a generic $U(\phi)$ the resulting effective potential vanishes when $V(\phi_0) + M = 0$ and it goes as $V_{eff} = \frac{1}{4U(\phi_0)}(V + M)^2$ in the region $V + M \sim 0$.

3.2 Including the gauge fields

The weak NGVE principle allows to incorporate gauge fields in a way which is simpler than in the context of the strong NGVE principle. Taking in Eqs. (7)

$$\begin{aligned} L_1 &= -\frac{1}{\kappa}R(\Gamma, g) + g^{\mu\nu}(\partial_\mu - ieA_\mu)\phi(\partial_\nu + ieA_\nu)\phi^* - V(\phi) \\ L_2 &= -\frac{1}{4}g^{\alpha\beta}g^{\mu\nu}F_{\alpha\mu}F_{\beta\nu} + \Lambda \end{aligned} \tag{19}$$

we see that except of the V-term in L_1 and Λ-term in L_2, the action (7) is invariant under the LES (notice that in S_2 the LES takes the form of the conformal transformation of the metric $g_{\mu\nu}(x) = J^{-1}(x)g'_{\mu\nu}(x)$). Therefore, as one can check explicitly, the gauge field does not enter in the constraint which turns out to be identical to Eq. (18).

For the gauge field in the unitary gauge we get $(-g)^{-1/2}\partial_\mu(\sqrt{-g}g^{\mu\alpha}g^{\nu\beta}F_{\alpha\beta}) + e^2\chi\varphi^2 g^{\mu\nu}A_\mu = 0$. In the Einstein conformal frame where the metric is $\overline{g}_{\mu\nu} = \chi g_{\mu\nu}$, the gauge field equation takes the canonical form

$$\frac{1}{\sqrt{-\overline{g}}}\partial_\mu(\sqrt{-\overline{g}}\;\overline{g}^{\mu\alpha}\overline{g}^{\nu\beta}F_{\alpha\beta}) + e^2\varphi^2\overline{g}^{\mu\nu}A_\mu = 0 \tag{20}$$

The gravitational and scalar field equations including the effective scalar field potential in the Einstein picture coincide with the appropriate equations of the model in the previous subsection 3.1. Keeping the assumption that $\Lambda > 0$ we can see that all conclusions concerning the vanishing of the vacuum energy in the true vacuum state $\varphi = \varphi_0$ where $V(\varphi_0) + M = 0$ remain unchanged. In addition to this, the Higgs mechanism for the mass generation in the true vacuum state φ_0 works now in a regular way in contrast to the model based on the strong NGVE principle where

the effective coupling constant (and as a consequence, the mass of the gauge boson) depends on the integration constant.

4 The true vacuum energy density as an effect of the SGMF symmetry breaking

In the previous section we have seen that the presence of the cosmological constant term in the original action of the weak NGVE theory does not change the result of the strong NGVE theory: the vacuum energy density of the TVS (in the Einstein picture) is zero. We are going to show now that the appearance of the nonzero vacuum energy density in the TVS (that is the effective cosmological term) in the Einstein picture can be the result of the explicit breaking of the SGMF symmetry (6) or (8) by adding the simplest form of a SGMF symmetry breaking term (9) to the action (7). Since such term is invariant under the LES, it does not contribute to the constraint as one can check explicitly, and therefore the constraint coincides with Eq. (18).

In this case the gravitational equations in the Einstein frame which still is defined by $\overline{g}_{\mu\nu} = \chi g_{\mu\nu}$ are

$$R_{\mu\nu}(\overline{g}) - \frac{1}{2}\overline{g}_{\mu\nu}R(\overline{g}) = \frac{\kappa}{2}\{\phi_{,\mu}\,\phi_{,\nu} - \frac{1}{2}\overline{g}_{\mu\nu}\phi_{,\alpha}\,\phi_{,\beta}\,\overline{g}^{\alpha\beta} + [\frac{1}{4\Lambda}(V+M)^2 + \lambda]\overline{g}_{\mu\nu}\} \qquad (21)$$

In the TVS $\phi = \phi_0$ where $V(\phi_0) + M = 0$ the last term in Eq. (21) acts as an effective cosmological term.

5 Model with continuous symmetry related to the NGVE principle and SSB without generating a massless scalar field

It is interesting to see what happens in the above models for the choice $V = J\phi$, where J is some constant. For simplicity we consider here the case (B) (see Sec. 3.1). Then the action (16) is invariant (up to the integral of total divergence) under the shift $\phi \to \phi + const$ which is in fact the symmetry $V \to V + const$ related to the NGVE principle. Notice that if we consider the model with complex scalar field ψ, where ϕ is the phase of ψ, then the symmetry $\phi \to \phi + const$ would be the $U(1)$ - symmetry.

The effective potential $V_{eff}(\phi) = \frac{1}{4\Lambda}[M + V(\phi)]^2$ in such a model has the form

$$V_{eff} = \frac{1}{2}m^2(\phi - \phi_0)^2 \tag{22}$$

where $\phi_0 = -M/J$ and $m^2 = J^2/2\Lambda$. We see that the symmetry $\phi \to \phi + const$ is spontaneously broken and mass generation is obtained. However, *no massless scalar field results from the process of SSB in this case, i.e. Goldstone theorem does not apply here.*

This seems to be a special feature of the NGVE - theory which allows: 1) To start with linear potential $J\phi$ without destroying the shift symmetry $\phi \to \phi + const$, present in the $\partial_\mu\phi\partial^\mu\phi$ piece, due to the coupling to the dynamical measure (4). This shift symmetry is now a symmetry of the action up to a total divergence. 2) This potential

gives rise to an effective potential $(M + J\phi)^2/4\Lambda$. The constant of integration M being responsible for the SSB.

In the case (C) (Sec.4) for the choice $V = J\phi$, the only difference is the appearance of the λ-term in addition to Eq.(22). Similar effect can be obtained also in the strong NGVE - theory (without introduction of an explicit Λ-term, as in Sec.2) but with the use of 4-index field strength condensate. The possibility of constructing spontaneously broken $U(1)$ models which do not lead to associated Goldstone bosons is of course of significant physical relevance. One may recall for example the famous $U(1)$ problem in QCD [13]. Also the possibility of mass generation for axions is of considerable interest. These issues will be developed further in elsewhere [14].

6 Discussion and Conclusions

We have seen that the consideration of a measure Φ independent of $\sqrt{-g}$, which in fact means that its relation to $g_{\mu\nu}$ and other fields is to be found dynamically, provides a new approach to the resolution of the CCP. In fact the consideration of this dynamical measure, in the context of the first order formalism solves the CCP in a wide range of models where the structures S_1 and S_2 of Eq.(7) are allowed to appear, thus ensuring that the SGMF symmetry (6) or (8) are satisfied.

The violation of the SGMF symmetry leads in contrast to an explicit appearance of an energy density in the TVS as we have seen in Sec.4 in a specific example. If this violation appears as a result of quantum corrections, it represents then the appearance of an anomaly of the SGMF symmetry (6) or (8). We then have a reason

for the smallness of such terms if not of their absolutte vanishing (in the case of exact symmetry). This resembles the situation of quark masses and chiral invariance (CI). In this case, as it is well known, CI forbids a quark mass. If a quark mass nevertheless appears, CI ensures that quark masses remain small even after the consideration of quantum corrections. In a similar way if a small SGMF symmetry breaking term appears, hopefully, it will not be renormalized into a large contribution after quantum effects are considered.

Since the main results of this work seem to depend on the existence of the SGMF symmetry, it is interesting to see the possible interpretations of this symmetry.

First of all notice that when $\Phi \neq 0$, $L_1 = const.$ and the symmetry (8) represents just a trivial shift of φ_a by a constant. This shift becomes nontrivial if we allow for points where $\Phi = 0$ and L_1 can vary in such "defect" points. Such defects can be naturally associated with extended objects and the nontrivial symmetry (8) appears then like invariance under reparametrization of the extended object coordinates φ_a.

A complete group-theoretical and/or algebraic study of the infinite dimensional symmetries of the theory has not been carried out yet and this should be an interesting exercise. One should notice that in addition to (8) there is the infinite dimensional symmetry of volume preserving internal diffeomorphisms (VPD) $\varphi'_a = \varphi'_a(\varphi_b)$ such that $\Phi' = \Phi$. Such transformations applied after a transformation of the form (8) lead to something new. The full symmetry group of transformation contains element that are not in SGMF and not in VPD which are only subgroups of the yet unknown full group of internal symmetries of the measure scalars φ_a.

In our treatment of the symmetry $L_1 \to L_1 + const.$ we have ignored possible topological effects, since $\int \Phi d^4x$ being a total divergence can be responsible however for topological effects, similar to the well known Θ-term in QCD.

As we have seen in Sec.5, such symmetry can be exploited to construct (through the introduction of a linear potential $V = J\phi$ coupled to the measure Φ) a theory that is globally $U(1)$-invariant with SSB and yet without appearance of a Goldstone boson. Possible applications to axion mass generation, etc has to be explored.

Finally, one can see that in the presence of matter which does not satisfy automatically the constraint (12), that is it is not LES-invariant, such matter will influence the effective potential of a scalar field in the Einstein picture, through the constraint. For example, including dust in L_1 of the model of Sec.3.1, it is found that the constraint links Λ to the amount of dust in the homogeneous cosmological solutions [14]

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